EPISODIC FUTURE THINKING IN MATHEMATICAL SITUATIONS

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Episodic future thinking is a process of mentally projecting one's self into a future event, allowing the event to be experienced before it actually occurs (Atance & O'Neill, 2001). The current study explores the possibility that students engage in episodic future thinking while solving mathematical tasks. Participating students were given mathematical situations and verbalized thoughts that emerged as they planned resolutions to the situations. All participants exhibited episodic future thinking and we present a categorization of these thoughts. Given extant results on the positive influence episodic future thinking has on general problem-solving ability, we propose that a similar influence might exist on mathematical problem solving.

INTRODUCTION

Many frameworks have been created to understand students' behaviours and thought processes that emerge when working with mathematics problems. Perhaps the most well-known are those in Schoenfeld's (1985) *Mathematical Problem Solving* and Pólya's (1945) *How to Solve It*; see (English & Sriraman, 2010) for a contemporary review. In both frameworks, "planning" is identified as an essential component of problem solving, or, more generally, while engaging with mathematical situations. Exactly how one should plan in a mathematical situation is not clear. Pólya's approach is in terms of *heuristics*, or general, situation-independent guidelines of how to act. Schoenfeld takes a different direction and introduces the notions of *resources* and *control*: problem solvers ought to draw on their acquired mathematical knowledge, knowing or determining what is relevant (resources) and what is extraneous (control). We take the view that neither perspective alone can account for the varied, idiosyncratic ways in which people actually navigate mathematical situations.

Episodic and Semantic Memory

Schoenfeld's conceptualization of resources as mathematical knowledge has been used in a restricted sense as one's understanding of mathematical concepts and procedures. Phrased in terms of cognitive structures, "knowledge", mathematical or otherwise, is a part of *declarative memory*: memories, facts, and experiences that can be consciously recalled and verbalized or written. Declarative memory can be partitioned into *episodic* and *semantic* memory systems (Tulving, 1983). Semantic memory is the memory of facts—knowing that Szeged is in Hungary, for example—whereas episodic memory is the memory of specific personal experiences—remembering the moment you learned from your mother that Szeged is in Hungary. In this light, we propose a more general conceptualization of mathematical knowledge as comprising both episodic and semantic memories of mathematics.

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When a student is engaged in learning any subject, including mathematics, they form both semantic and episodic memories; not only do they acquire knowledge of facts, definitions, procedures, and general processes, they also form memories of personal experiences of learning and using these facts and definitions, executing procedures, and engaging in mathematical thinking. Episodic memories of engaging with mathematics are not often reported in the literature, and typically viewed as irrelevant to the mathematics. We take the view that both semantic and episodic memories are vital to working in mathematical situations. The reliance on episodic memory may be particularly evident for mathematical situations that require some degree of planning, given recent evidence that episodic memory contributes to future thinking (Schacter, Addis & Buckner, 2007; Schacter et al., 2012).

Episodic Future Thinking

Mathematics aside, what is known about how people plan for to-be-experienced events? Take, for example, planning a trip to Szeged. One may form a list of what to pack using only semantic knowledge—I will be away for ten days and therefore require ten undershirts, etc. One may also imagine themselves travelling to Szeged, even having never travelled there, and can form memories of these imagined future events. These simulations of the future, of experiencing the expected weather or being at the conference dinner or reconnecting with colleagues, can be used to plan for the trip. This pre-experiencing of the event may help one anticipate how the event will unfold, including what could go wrong and what may be required. The key point here is that the person does not necessarily plan for the to-be-experienced event semantically; they may also plan episodically. Indeed, this type of pre-experiencing of a future event, called *episodic future thinking* (Atance & O'Neill, 2001), is associated with more successful outcomes in various goal-directed activities including open-ended problem-solving (Taylor, Pham, Rivkin, & Armor, 1998; Schacter, 2012).

This paper explores the possibility that students engage in episodic future thinking when planning in mathematical situations. Participating student volunteers were given, sequentially, two mathematical situations and asked to think about what a solution could be. After mentally forming a solution, participants were interviewed about the thoughts and mental images they experienced when forming their solution. All students engaged, to varying degrees, in episodic future thinking.

The purpose of this paper is twofold: to 1) present evidence of people engaging in episodic future thinking while working in mathematical situations, and, 2) to highlight a number of methodological issues that must be addressed to further explore episodic future thinking in mathematics. This work is a part of a larger study intended to create a theory of future thinking in mathematics.

METHODS

Student volunteers were recruited, via email and an in-class announcement, from a second year general mathematics course, covering multivariable calculus and elements of linear algebra, at the University of Auckland during the second semester. A total of nine students volunteered from this course. The recruitment email was sent

Task 1

Graph a function $h(x)$ that satisfies the following conditions.

- $h(0) = 2$
- $h'(x) > 0$ when $x < -1$, $h'(x) < 0$ when $x > -1$
- $h''(x) > 0$ when $x < -2$ and when $x > 0$, $h''(x) < 0$ when $-2 < x < 0$
- $\lim_{x \to 0} h'(x) = \infty$

Imagine the graph of this function to the best of your ability and in as much detail as possible.

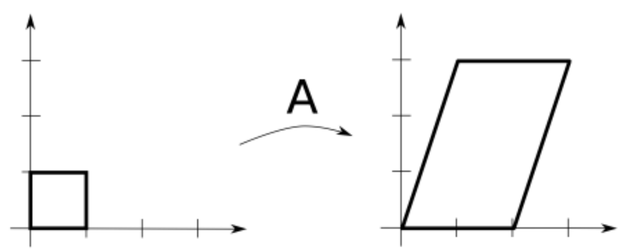
Task 2

We are becoming familiar with the sight of algal blooms on our beaches in the Summer. We are told that these blooms are more frequent as a result of a number of factors, including warmer sea temperatures and greater nutrient concentrations. We observe that the algae congregates in certain places on our beaches and comes and goes over a period of days and weeks.

Your task is to construct a mathematical model of an algal bloom. That is, a mathematical description of the algal bloom that descibes how it changes over time. Imagine this model to the best of your ability and in as much detail as possible.

Task 3

Suppose the matrix A maps the unit square in $\mathbb{R}^2$ to the parallelogram in the right half of the figure below.



What are the eigenvalues of A^{-1}?

Task 4

Suppose you work on a road construction site where fuel for all the machines is stored in a large cylindrical container. This cylinder is lying on its side. There is a small opening halfway along the tank facing upwards in which a measuring stick can be inserted to find the distance between the surface of fuel and the opening. What is an expression for the amount of fuel in the cylinder?

Figure 1: The mathematical tasks used in this study.

to an honours version of this course, yielding an additional two participants, for a total of eleven ($N = 11$). Following informed consent, each student participant individually attended two sessions, each approximately one-half hour in duration, and was compensated with a university bookstore gift voucher for each session they attended. In the first session, they were asked to verbalize their thoughts about given mathematical topics. The results of this first set of interviews are reported in a companion manuscript. The focus of the current is on the second session during which students were given, sequentially, two mathematical situations from the four presented in Figure 1, and asked to think about how they would solve each. They then verbalized their thinking with prompts from the interviewer.

Students' interviews were audio recorded and transcribed. The transcripts were analyzed using a Grounded Theory methodology (Strauss & Corbin, 2015). The initial analysis consisted of classifying the students' utterances as: 1) *episodic*, relating to events experienced, or to-be experienced, by the participant, 2) *semantic*, comprising factual statements, and 3) *other*, typically clarification or orientation statements. The episodic category was further refined into episodic memories—which naturally have occurred in the past—and episodic future thinking. Only those student utterances indicative of episodic future thinking are reported here. Following Tulving (1983) and Atance and O'Neill (2001), for an utterance to be classified as an episodic future thought, it must 1) refer to an event that is forecast to occur at some future time, and 2) involve the speaker.

RESULTS

All participants engaged in some form of episodic future thinking while thinking about the provided mathematical situations. Episodic future thinking is as idiosyncratic as the participants' personal histories, making an authoritative categorization elusive. There were nevertheless similarities among some of the participants' utterances and these informed the creation of the descriptive categories reported below. Each category applies to the utterances of at least two participants. Representative quotes are provided, identified by participant numbers of the form Px.

Imagining Possible Actions (PA)

This type of mental simulation involved anticipating what actions would need to be taken to complete the mathematical task. Participants described what they would need to write or operations they would perform to make progress on the task.

> "So it's just sitting there trying to think about what you would do, what would be required, and how to piece all those little bits of information of what would be required together to get a correct solution. And then checking it back to make sure that that's actually what you've got."(P2)

> "So if I was to, like, flesh this out, I would write it in terms of 'n'."(P1)

Responses in this category were not restricted to participants thinking about what to write or what methods to use, however. One participant, while recalling their thoughts about Task 1, indicated they thought about what movements their hand would need to take to form the graph.

> "I was thinking of how to write it...what I imagine is a pen in hand...how to draw the graph."(P8)

Imagining Future Social Situations (SS)

A couple of participants imagined working the mathematical tasks in social settings.

> "I imagined working with friends though...because I don't know how to do it. So I assume some of my friends would know how to do it...sitting around a circled desk...I would get this far and then they would help me along the way to find the formula."(P0)

> "I kind of just thought how I would, like, teach this to someone who didn't know what any of this meant...and I think before I started saying anything I just sort of planned out the structure of what I was going to say."(P1)

Adapting a Past Experience (PE)

A few of the participants recalled past events of working with similar mathematics. But these were not strict recollections of previous events; they were adapted to the context of the current mathematical situation.

> "My first thought was to use triangles, because in the last year of school...we used to use triangles a lot in rates of change questions."(P10)

> "In my tutorial we had eigenvectors. They were given like this...but I couldn't remember what I did with the eigenvectors...I think you need to use one of them...sub it into this."(P0)

One participant, while recalling their thoughts about Task 2, described their mental image of an expanding lily pad. Initially, the interviewer assumed this was spontaneously generated by the problem statement. The image was brought up again later in the interview and was revealed to be an actual problem the participant had previously encountered and was trying to modify to Task 2.

Experiencing Emotions (EE)

Some students experienced emotions they associated with resolving the mathematical tasks. These were either inhibitory or conducive to engaging with the tasks.

> "So I was just imagining myself in a room full of people with a test script in front of me and just freaking out a bit because it looks so hard. But also kind of confident because I know I've done problems like these before and I know I've always...got them right. So even though I'm looking at...no idea where to start, I know that I've felt like that before but I've always solved it."(P1)

> "I think about the endpoint. I think about...like the satisfaction feeling when you actually solve the problem."(P11)

Anticipating Failure (AF)

The final category of episodic future thoughts experienced by the participants involved them imagining working the mathematical task but being unable to make progress.

> "I can recognize the components. And I can recognize how I might want them in there. I can't write them. So I was sitting here thinking, I really don't want to have to write this out properly because I don't know how to. ... you just think of all these things that you might have to do that you won't be able to do."(P2)

"...I just pictured myself in a test room...or an exam room and just sort of like doing the first derivative test...I felt nervous, because I couldn't do it." (P6)

Other Types of Episodic Future Thinking

The participants verbalized an assortment of episodic future thoughts, such as: imagining what information and/or external resources they would need to draw on while forming the solution, mentally simulating the situation given in Tasks 2 and 4, modifying heuristics to the given task, and imagining a virtual path they would need to traverse to arrive at a solution. Each of these thoughts were verbalized by at most one participant, however, which lends support to the claim made above that episodic future thinking is highly idiosyncratic.

CONCLUSIONS AND FUTURE THOUGHTS

This paper presents evidence that undergraduate students currently enrolled in a mathematics course do engage in episodic future thinking while working with mathematics. Episodic future thinking is just one possible process in which a student may engage and there are, of course, others: they may think of and apply the appropriate problem schema, or they may not be able to think mathematically at all. We present here a number of issues that ought to be addressed in order to further investigate episodic future thinking in mathematics.

The first is that episodic future thinking may not always arise in mathematical work—for example, when adding two single-digit numbers—and we are left wondering about the type of mathematical situation likely to prompt episodic future thinking. We argue, as in (Maciejewski & Barton, 2015), that the mental processes evoked depends on how the task relates to its solver. As Schoenfeld (1985) recognizes, a mathematical problem is only a *problem* if it relates to the solver in that way. If a task is too familiar to its solver, automaticity may be triggered and no thinking evoked. If the task is too unfamiliar to the solver, they may not be able to understand it, let alone make any progress in its resolution. Familiar, but not too familiar, tasks may invoke a problem schema. We suspect that unfamiliar-yet-understandable tasks, like the "problems" of Schoenfeld (1985), are the realm of those which elicit episodic future thinking. In such tasks, a solution, not readily apparent, must be consciously constructed and planned for; see, for example, (Lesh & Zawojewski, 2007). This notion of a *distance* between a problem and its solver, as proposed in (Maciejewski & Barton, 2015), appears new and may provide further insight into the genesis of the solver's actions. With this in mind, we are left desiring a more complete description of how students and problems might relate.

The second issue is the interaction between episodic future thinking and success in mathematical situations. Specifically, are students who are more able to engage in episodic future thinking, perhaps by forming more lucid mental simulations, better problem solvers? Previous work outside of mathematics does support the claim that more effective episodic future thinkers are more effective problem solvers (Taylor, Pham, Rivkin & Armor, 1998; Schacter, 2012) but we are not aware of any work that considers discipline-specific problem-solving. We are also left wondering about the converse relationship: do effective mathematics problem solvers engage in episodic future thinking in their mathematical work? There is some affirmative evidence from mathematicians (Maciejewski & Barton, 2015); might the same be true for students?

The third issue concerns whether episodic future thinking in mathematics can be developed through training. As reviewed by Taylor, et al. (1998), psychology students who were trained to imagine the *process* of attaining an academic goal outperformed those with no training. In fact, those who were trained to imagine the *outcome* of attaining the goal (e.g., feeling relief or accomplishment) achieved at an even lower level. These results demonstrate that the development of episodic future thinking can, in some domains, be aided with an instructional intervention. We are optimistic that a similar result can be achieved in mathematics. Additionally, the results reported in (Taylor, et al., 1998) suggest that imagining a to-be-experienced event allows one to “pre-experience” the emotions associated with that event—as did the students in the *EE* category above—rendering one better able to manage those emotions if they do arise in the actual event. Such results are likely to have profound implications for students paralyzed by mathematical anxiety.

A number of methodological issues arose when planning, conducting, and interpreting the interviews in this study. The first, mentioned above, was the selection of the tasks. Of all the participants in this study, only one was able to completely resolve any of the tasks. All others made no or only slight progress. We are left wondering what type of episodic future thoughts would have emerged if more students were able to successfully engage with more of the tasks. Developing tasks that relate to students in this way is non-trivial: it requires an understanding of a student's personal mathematical history and their current mathematical understanding. It would seem, therefore, a futile endeavour. We suggest addressing this issue from another angle: have the students develop, or choose, their own tasks. This may mimic the process mathematicians undertake when choosing a research-level problem: they often choose problems they think they are able to contribute to, or are likely to lead to aesthetically pleasing resolutions, or be of interest to their colleagues—all of these choices involve some aspect of episodic future thinking.

The second methodological issue arose during the interviews. The participants appeared to be *primed* to respond “mathematically”. Simply stating that the task is mathematical appeared, to the interviewers, to prompt the participants into talking strictly about mathematics rather than describing any and all thoughts they were experiencing. As one participant said, “I pretty much just constrained myself to doing the question, nothing outside the question because those are really quite irrelevant to the question.” Therefore, we see a need for more naturalistic ways of observing students in problem-solving situations.

Finally, we address an anticipated criticism of the current work. The episodic future thoughts articulated by the participants contained, more often than not, scant mathematics. Therefore, it could be argued that episodic future thinking may not actually be relevant to working with mathematics. These thoughts were nevertheless experienced by our participants and we expect they are experienced more widely. Moreover, the thoughts, mathematical or otherwise, verbalized by the participants guided the mathematical choices they made while working the tasks. This leads us to consider possible implications for educational practice. Two are immediately apparent: instructors/teachers may 1) guide students to suppress or disregard seemingly irrelevant thoughts emerging while engaged with mathematics and revert to a "control/resource" approach similar to Schoenfeld's (1985), or 2) acknowledge that students might experience episodic future thoughts and assist them in making these vivid and productive. Since this paper is among the first that explore students' episodic future thinking in mathematical situations, it is too early to form concrete recommendations for educators. Nevertheless, we foresee training students to develop and hone their episodic future thinking in mathematical situations a productive way of significantly improving the efficacy of their mathematical problem solving.

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